Faraday's Law of Induction

10.1 Faraday's Law of Induction

The electric fields and magnetic fields considered up to now have been produced by stationary charges and moving charges (currents), respectively. Imposing an electric field on a conductor gives rise to a current which in turn generates a magnetic field. One could then inquire whether or not an electric field could be produced by a magnetic field. In 1831, Michael Faraday discovered that, by varying magnetic field with time, an electric field could be generated. The phenomenon is known as electromagnetic induction. Figure 10.1.1 illustrates one of Faraday's experiments.

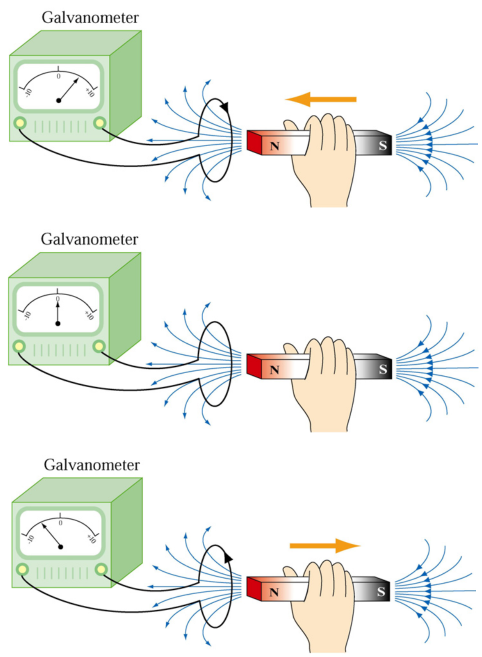

Figure 10.1.1 Electromagnetic induction

Faraday showed that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop. However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop. In particular, the galvanometer deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away.

Faraday's experiment demonstrates that an electric current is induced in the loop by changing the magnetic field. The coil behaves as if it were connected to an emf source. Experimentally it is found that the induced emf depends on the rate of change of magnetic flux through the coil.

10.1.1 Magnetic Flux

Consider a uniform magnetic field passing through a surface *S*, as shown in Figure 10.1.2 below:

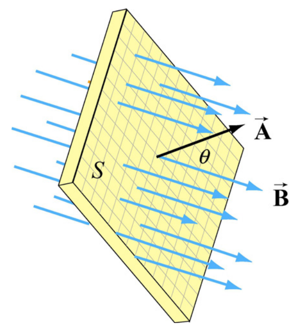

Figure 10.1.2 Magnetic flux through a surface

Let the area vector be $\vec{\mathbf{A}} = A\hat{\mathbf{n}}$, where A is the area of the surface and $\hat{\mathbf{n}}$ its unit normal. The magnetic flux through the surface is given by

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta \tag{10.1.1}$$

where θ is the angle between $\vec{\mathbf{B}}$ and $\hat{\mathbf{n}}$. If the field is non-uniform, Φ_B then becomes

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \tag{10.1.2}$$

The SI unit of magnetic flux is the weber (Wb):

$$1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2$$

Faraday's law of induction may be stated as follows:

The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt} \tag{10.1.3}$$

For a coil that consists of *N* loops, the total induced emf would be *N* times as large:

$$\varepsilon = -N\frac{d\Phi_B}{dt} \tag{10.1.4}$$

Combining Eqs. (10.1.3) and (10.1.1), we obtain, for a spatially uniform field $\vec{\mathbf{B}}$,

$$\varepsilon = -\frac{d}{dt}(BA\cos\theta) = -\left(\frac{dB}{dt}\right)A\cos\theta - B\left(\frac{dA}{dt}\right)\cos\theta + BA\sin\theta\left(\frac{d\theta}{dt}\right) \qquad (10.1.5)$$

Thus, we see that an emf may be induced in the following ways:

(i) by varying the magnitude of $\vec{\mathbf{B}}$ with time (illustrated in Figure 10.1.3.)

Figure 10.1.3 Inducing emf by varying the magnetic field strength

(ii) by varying the magnitude of $\vec{\mathbf{A}}$, i.e., the area enclosed by the loop with time (illustrated in Figure 10.1.4.)

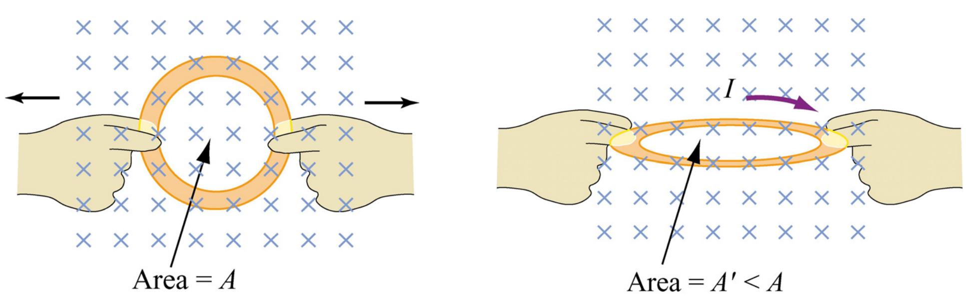

Figure 10.1.4 Inducing emf by changing the area of the loop

(iii) varying the angle between $\vec{\mathbf{B}}$ and the area vector $\vec{\mathbf{A}}$ with time (illustrated in Figure 10.1.5.)

Figure 10.1.5 Inducing emf by varying the angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$.

10.1.2 Lenz's Law

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{\mathbf{A}}$.

2. Assuming that $\vec{\mathbf{B}}$ is uniform, take the dot product of $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt}: \begin{cases} > 0 & \Rightarrow \text{ induced emf } \varepsilon < 0 \\ < 0 & \Rightarrow \text{ induced emf } \varepsilon > 0 \\ = 0 & \Rightarrow \text{ induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{\mathbf{A}}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure 10.1.6.

Figure 10.1.6 Determination of the direction of induced current by the right-hand rule

In Figure 10.1.7 we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current I.